Written Exam Economics Winter 2018-19

Auctions: Theory and Practice

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This exam question consists of 5 pages in total (including this front page)

Answers only in English.

The paper must be uploaded as <u>one PDF document</u>. The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Digital Exam.

Introduction

Throughout the assignment, please show your work. Simply stating the correct answer without sufficiently explaining your calculations/reasoning is not enough to get full credit. Correspondingly, an incorrect answer that uses some of the correct argumentation may be given partial credit.

If you believe that there may be a typo in one of the questions, or if something is stated unclearly, please let us know as quickly as possible by sending an email to both Neil and Holger. Any material responses to such queries will be published on Absalon.

Good luck!

Problem 1 (True or false)

Please state whether each of the following statements is true or false and show the arguments and/or calculations which justify your conclusion.

1a. A "clock"-style English multi-unit auction is associated with exposure risk for bidders that have complementarity in their valuations. (a qualitative explanation is sufficient)

- False.
- Bidders that have complementarity in their valuations may, at a certain price per unit, be interested in acquiring several units, but not a subset of those units. Exposure risk arises in auction formats that treat a bidder's bid for several units as being separable into bids for individual units, meaning that a bidder can be awarded fewer units than their total bid (at a price potentially exceeding their valuation for the subset of units – this is the "risk").
- In a "clock"-style English multi-unit auction, bidders bid for a "package" of units in each auction round and the auction format thus does not treat a bidder's bid for several units as being separable into bids for individual units. Hence, a clock-style auction is not associated with exposure risk.

1b. First-price sealed-bid auctions are more susceptible to collusion than second-price sealed-bid auctions. (a qualitative explanation is sufficient)

- False.
- First-price auctions are less susceptible to collusion than second-price auctions because in a first-price auction, ring members who have agreed to suppress their bids have an incentive to deviate from the agreement (whereas in a second-price auction, ring members have no incentive to deviate).
- This means that bidding rings are less likely to arise in first-price auctions (as ring members will not be able to trust one another) – and first-price auctions are thus less susceptible to collusion.

1c. The presence of a resale market following a discriminatory sealed-bid auction will ensure efficiency. (a qualitative explanation is sufficient)

- False.

- Generally, discriminatory multi-unit auctions do not allocate objects efficiently (Krishna: Proposition 13.7) since bidding strategies are not separable and symmetric across units – bidders will shade more on the additional units than on the first unit. This is analogous to the result that first-price auctions with asymmetric bidders are not efficient.
- The presence of a resale market following a discriminatory sealed-bid auction will not ensure efficiency. This is because – analogous to the result regarding resale markets following a first-price auction with asymmetric bidders – bidders will adjust their bidding strategies in anticipation of a resale market, e.g. high-value bidders will bid less aggressively to avoid revealing their true values. The winning bidder(s) will thereby not be able to accurately guess the true value of potential buyers on the resale market, which creates a matching problem on the resale market that will impede an efficient reallocation of the objects.

1d. The expected revenue of a first-price sealed-bid auction with 2 bidders whose private values are uniformly distributed between 0 and 8, with a reserve price of 2, is 4.

- False.
- We can calculate expected revenue for this first-price auction by calculating expected revenue for a second-price sealed-bid auction with the same characteristics – because we know that the revenue equivalence theorem applies (so the calculation for a second-price auction must yield the same result as if we had calculated for a first-price auction). We do this because it is easier to calculate expected revenue for a second-price auction.
- We calculate expected revenue in a second-price auction with a reserve price and 2 bidders by summing revenue across 3 cases, weighted by the probability of each case occurring:
    1. Both bidders draw below r – revenue is 0
    2. Exactly one bidder draws above r – revenue is r
    3. Both bidders draw above r – expected revenue is equivalent to the expected second highest value
- Case 1 contributes to total expected revenue with exactly 0.
- Case 2 occurs with probability $F(r) * (1 - F(r)) * 2$ since exactly one of the two bidders must draw below the reserve price, exactly one must draw above, and this can occur in two ways (either the first bidder draws above and the second one draws below, or vice-versa). This probability can be calculated as:
$$F(r) * (1 - F(r)) * 2 = F(2) * (1 - F(2)) * 2 = \left(\frac{2}{8-0}\right) * \left(1 - \frac{2}{8-0}\right) * 2 = \frac{1}{4} * \frac{3}{4} * 2 = \frac{6}{16} = \frac{3}{8}$$
Revenue in this case is equal to the reserve price, which is 2. This means that the contribution of case 2 to total expected revenue, weighted by probability, is $\frac{3}{8} * 2 = \frac{6}{8}$.
- Case 3 occurs with probability $(1 - F(r))^2$, since both of the two bidders must draw above the reserve price. This probability can be calculated as:
$$(1 - F(r))^2 = (1 - F(2))^2 = \left(1 - \frac{2}{8-0}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$
Revenue in this case is the expected second highest draw. We know that the expectation of the kth-highest draw among N bidders that are uniformly distributed between $\underline{x}$ and $\overline{x}$ is $\underline{x} + \frac{N+1-k}{N+1}(\overline{x} - \underline{x})$. In this case, we have 2 bidders distributed between r and $\overline{x}$ (since we are considering the case where both have drawn above r), and we are interested in the expectation of the second-highest draw:
$$\underline{x} + \frac{N+1-k}{N+1}(\overline{x} - \underline{x}) = 2 + \frac{2+1-2}{2+1}(8-2) = 2 + \frac{1}{3} * 6 = 4$$
This means that the contribution of case 3 to total expected revenue, weighted by probability, is $\frac{9}{16} * 4 = \frac{9*4}{4*4} = \frac{9}{4}$.
- Total expected revenue is the sum of the three cases: $0 + \frac{6}{8} + \frac{9}{4} = \frac{6}{8} + \frac{18}{8} = \frac{24}{8} = 3$, which is not 4.

1e. A common value auction with N symmetric bidders and a diffuse prior common value component, v, where each bidder receives a signal, x_i, independently drawn from a uniform distribution on $[v - ½, v + ½]$, has a symmetric bidding strategy in a sealed-bid second-price auction equal to $b_i = x_i + ½ - (\frac{N-1}{N})$.

- True.
- The symmetric equilibrium bidding strategy in a common value (affiliated signals) 2nd-price auction is given by $\beta^{II}(x) = v(x, x)$ meaning that bidders bid the expected (common) value conditional on winning the auction by tying with one other bidder.
- Each bidder will infer that, by winning the auction, he or she will have the highest signal among N-1 bidders. Since signals are uniformly distributed on the interval $[v - ½, v + ½]$, this expectation is equal to:

$$x_i = v - ½ - \frac{N-1+1-1}{N-1} - (v + ½ - (v - ½))$$

$$\begin{aligned} x_i &= v - ½ + \frac{N-1+1-1}{N-1+1}(v + ½ - (v - ½)) \\ &= v - ½ + \frac{N-1}{N} \end{aligned}$$

Which we can solve for v to give us the expected value and thereby the symmetric bidding strategy:

$$b_i = v = x_i + ½ - \frac{N-1}{N}$$

Problem 2

Lobbying, where different interest groups compete to influence politicians, can be thought of as a type of all-pay auction. All lobbyists (i.e. the bidders) have a private value for a specific policy – e.g. lowering the corporate tax rate – and will put some effort and funds into a lobbying campaign (i.e. a bid) to tilt policy in its favoured direction. Only some lobbyists will be successful and receive a "pay-off" in the form of a concrete policy change.

Consider N risk-neutral lobbyists with valuations x independently and uniformly distributed between 0 and 1.

2a. Find the equilibrium bidding strategy for N lobbyists in a sealed-bid all-pay auction for a single unit (policy). Please also explain the intuition of your result.

- Assuming the existence of a symmetric equilibrium with an equilibrium bidding strategy $\beta^{AP}(x)$, we know that we can formulate the expected payment in an all-pay auction as: $m_i^{AP}(x) = \beta^{AP}(x_i)$. This is because, in an all-pay auction, the expected payment does not depend on the probability of winning – the payment always materialises.
- We know that the revenue equivalence theorem equates expected revenue in an all-pay auction with expected revenue in e.g. a second-price auction. So, expected payments for a bidder with the same valuation must be the same in both auction types.
- We know that bidders in a second-price auction have an expected payment which is: $m_i^{II}(x) = prob(i\ wins|x) * E(payment|i\ wins, x) = prob(i\ wins|x) * E(\beta^{II}(Y_1)|i\ wins, x)$
- The revenue equivalence theorem means we can equate the two payments, i.e.: $m_i^{AP}(x) = m_i^{II}(x_i) = \beta^{AP}(x_i) = prob(i\ wins|x) * E(\beta^{II}(Y_1)|i\ wins, x)$
- We know that the expectation of Y_1, conditional on winning (for a uniform distribution between 0 and 1 with N bidders), is: $E(Y_1) = \underline{x} + \frac{N-1}{N}(x - \underline{x}) = \frac{N-1}{N}x$

- We also know that the probability of i winning, conditional on drawing x, in a symmetric equilibrium, is equal to the probability that all N-1 other bidders make draws below x. Because all of the other draws are independent, this can be written as: $F(x)^{N-1}$. For uniform distributions between 0 and 1, this is equivalent to: x^{N-1}
- Inserting these results gives us:
$$\beta^{AP}(x_i) = x_i^{N-1} * \frac{N-1}{N} x_i = x_i^N * \frac{N-1}{N}$$
Which is the result we are looking for.
- Note: using the assumption that $\beta^{AP}(x_i) = \alpha^{AP} * x_i$ will also find the right result – however since α^{AP} in fact depends on x_i, it is not a constant, and this method is thus technically incorrect.
- Intuition: The bidders shade in equilibrium since they will pay their bid regardless of whether they win the item or not and it is thus optimal to shade more than the equivalent equilibrium bidding strategy in a first-price auction where only the winner pays his bid.
- The optimal degree of shading decreases in the signal, x. Bidders with a low signal will infer that it will be very unlikely that they have the highest signal (thereby winning the auction) and it is will thus be optimal for these bidders to shade a lot to minimise their payment. Conversely, bidders with a higher signal are willing to shade less since they have a higher probability of winning.
- The relationship between the optimal degree of shading and the number of bidders, N, depends on two opposite effects:
    - On the one hand, bidders will shade *less* with a higher number of bidders since they will have a lower chance of winning and will be willing to give up some payoff to increase their chance of winning (the same effect as in a first-price auctions). For a high signal (above 0.75) and low N, this effect will dominate.
    - On the other hand, bidders will shade *more* with a higher number if bidders because their overall chance of winning is always lower. For signals below 0.75, this effect will always dominate and for $N \to \infty$ this effect will dominate, unless for bidders with a signal of 1.

2b. Is this auction efficient? (a qualitative explanation is sufficient)

- Yes, the auction is efficient in a symmetric equilibrium since the bidder with the highest value will always submit the highest bid and win.

Some lobbyists will face challenges when collecting funds for their lobbying campaign and will thus be budget constrained.

Now consider a specific all-pay auction with 3 bidders where 1 bidder has a budget constraint of $\frac{2}{3}$.

2c. Is this auction efficient? (a qualitative explanation is sufficient)

- Since the highest possible draw is 1, and since this would result in a bid of $\beta^{AP}(x_i) = \beta^{AP}(1) = 1^N * \frac{N-1}{N} = 1^3 * \frac{3-1}{3} = \frac{2}{3}$ according to the symmetric equilibrium bidding strategy, the highest-value bidder will never be budget-constrained. Because there are no *de facto* budget constraints, the auction is still efficient.

Problem 3

Danish farmer Jens Hansen has found 5 archaeological artefacts "Guldhorn", or Golden Horns, in his field. The Horns are in pristine condition and are all identical. The Horns are so-called "Danefæ" and thus property of the Danish state. After a lengthy debate, it has been politically decided that the Horns should be allocated to Danish museums.

The main policy objective for the allocation is that the Horns should be allocated to the museums that value them the most. The politicians are not concerned with extracting revenue from the museums or with ensuring an even distribution of the Horns between the different museums (i.e. it would not be a problem if one museum were to get all the Horns).

The Danish Ministry of Culture (the Ministry) has been tasked to find the optimal mechanism to allocate the Horns and has decided to use an auction.

3a. Assuming that the various museums have private valuations for the Horns, which auction format would you recommend and why? (a qualitative explanation is sufficient)

- The main policy objective is efficiency. We would thus recommend a Vickrey multi-unit auction format since this is the only multi-unit auction format that results in efficient allocations. There is no need to facilitate price discovery by implementing a multi-round format since values are private – so we would recommend a sealed-bid format.

Regardless of your recommendation, the Ministry decides to employ a Vickrey sealed-bid multi-unit auction.

The Ministry expects 3 museums to participate in the auction with the following marginal values:

Marginal values	**1st horn**	**2nd horn**	**3rd horn**	**4th horn**	**5th horn**
The Royal Museum	50	38	30	20	5
The Museum of Øster Hurup	60	15	10	5	2
Old-is-more (OIM)	45	32	10	5	1

3b. Find the equilibrium bids, allocation and payments. Explain qualitatively why no bidder has an incentive to deviate from their equilibrium bidding strategy.

- In the Vickrey auction it is a dominant strategy for bidders to bid their true marginal values. Intuitively, this arises since the winning bids do not determine the payments:
  - It can never be optimal for a bidder to bid *higher* than the true marginal values since this can only give a lower or unchanged payoff. If the bidder overbids but wins the same units as by bidding truthfully, the expected payment is unchanged. If the bidder overbids but wins an additional unit (or units), then the marginal payoff from this unit (or units) must be negative since the payment must be higher than the marginal value (as the bidder has only won the additional unit(s) by "pushing out" a higher bid, thereby paying that bid for the unit)
  - It can never be optimal for a bidder to bid *lower* than the true marginal value since this can only give a lower or unchanged payoff. If the bidder underbids but wins the same units as by bidding truthfully, the payment is unchanged. If the bidder underbids but foregoes winning a unit (or units), the bidder foregoes winning a unit with a non-negative payoff thereby giving a lower payoff.
- The equilibrium bids are therefore equal to the marginal values outlined in the table.
- Given the equilibrium bids, the equilibrium allocation of the 5 horns is:
  - TRM: 2 horns

    - MØH: 1 horn
    - OIM: 2 horns
- And the equilibrium payments are:
    - TRM: 15+10=25
    - MØH: 30
    - OIM: 30+20=50

Several of the government officials in the Ministry oppose the Vickrey format as the museums can end up paying different prices for otherwise identical Horns. One official proposes that the Ministry employs a uniform sealed-bid auction instead, since he has heard that this format will always result in the same allocation as the Vickrey format but will ensure the same price for all Horns.

3c. Is the official correct? Please also explain the intuition behind your answer. (a qualitative explanation is sufficient)

- No, the official is not correct. The uniform auction will, by definition, result in equal payments for the horns but will not guarantee an efficient allocation (i.e. not necessarily the same allocation as the Vickrey auction).
- The symmetric equilibrium bidding strategy in a uniform auction is to bid truthfully on the first unit, but shade bids on additional units (so-called demand reduction). Demand reduction arises because the bids on the additional units will, with some likelihood, become price-setting and may thereby determine the pay-off of the first unit.
- Due to this demand reduction (i.e. bidding below marginal values on later units), the equilibrium bidding strategy is not symmetric across units – which can lead to inefficient allocations. A bidder with a high marginal valuation for the 2nd Horn might shade so much that the Horn instead goes to a bidder with a lower marginal valuation for the 1st Horn (that submits a truthful bid for the 1st Horn).

Regardless of your input, the Ministry decides to pursue with the Vickrey sealed-bid multi-unit auction.

The Ministry suspects that it might not have the full overview regarding the number of museums that will bid in the auction and their marginal values. The Ministry has therefore hired a consultancy firm, Auctions Advice (AA), to give their best estimate of the number of bidders and their values.

AA has identified the same marginal values for the three museums listed above, but has also identified a fourth museum, Old Rocks, with the following marginal values:

Marginal values	**1st horn**	**2nd horn**	**3rd horn**	**4th horn**	**5th horn**
Old Rocks	10	70	0	0	0

3d. Given the marginal values displayed in the table above, which considerations would Old Rocks have in relation to its bidding strategy in the Vickrey auction? (a qualitative explanation is sufficient)

- Old Rocks has complementarity in its valuations and will face exposure risk from bidding in a standard Vickrey auction (as a standard sealed-bid auction treats bids as being separable into bids for individual units). In a standard Vickrey auction, Old Rocks might not bid higher than 10 for each unit as it would not want to risk ending up with only one Horn at a price higher than 10 and thus a negative payoff. It would thus be difficult for Old Rocks to reflect their true willingness to pay for two units (which is 70+10=80) in a standard Vickrey auction.

Now that they have discovered that Old Rocks may also participate in the auction, the Ministry is wondering whether a Vickrey auction is still the right choice.

3e. Would you propose any changes to the format? Please also explain the intuition behind your answers. (a qualitative explanation is sufficient)

- The Vickrey format can be augmented to allow for package bidding (this is called a "combinatorial" Vickrey auction). Old Rocks would thereby be able to submit two package bids: one horn for 10 or two horns for 80. We would recommend using a combinatorial Vickrey auction.

Another group of government officials are worried that the museums have an interest in increasing their competitors' payments for the horns.

3f. If museums were interested not only in maximising their own payoff, but also in minimising the payoffs of competing museums, how would this impact their bidding incentives in the Vickrey auction? (a qualitative explanation is sufficient)

- Due to the "2nd-price" element of a Vickrey price rule, bidders can affect the payments of competing winners' and thereby their payoffs. This can give rise to so-called "push bidding" whereby bidders "overbid" for units that they are sure they will not win – with the purpose of increasing the competitors' payments.
- Given the valuations in the table, MØH might, for example, have an incentive to overbid on the 2nd and 3rd Horns because its valuations for these additional horns are low (and because MØH might thus be certain that they will not win with a slightly higher bid). However, the strategy would not be without risk since MØH might end up winning the additional units at a loss if it has overestimated the competitors' valuations.

Problem 4

The British government is auctioning off two licenses for seabed rights in the North Sea to prospective offshore wind developers. The two licenses will be sold in two sequential second-price auctions.

The two licenses are for geographically separate regions, but the two regions are of exactly the same size and are located right next to each other, so any buyer would be indifferent between the two licenses.

There are three offshore wind developers that will submit bids for the licenses: DING, Waterfall and Equisyd. None of the bidders are budget-constrained and all three bidders are risk-neutral. Each of the developers is interested in acquiring only a single license, i.e. they have single-unit demand.

We assume that the three bidders can accurately calculate their own private value of acquiring a license, and that their values are independently and uniformly distributed between 1 and 2 million pounds.

4a. Find the equilibrium bidding strategy in each of the two sequential auctions. Please also explain the intuition of your result.

- We know that the equilibrium bidding strategy in the second auction will be to bid $\beta_2(x) = x$, because this is essentially a standard second-price auction.

- In the first auction, we can use the result that we know that bidders in sequential second-price auctions always bid as they would do in the subsequent auction in a series of first-price auctions. So, for the penultimate second-price auction, bidders bid as they would do in the last of a series of first-price auctions. In the last of a series of first-price auctions where the number of bidders only exceeds the number of auctions in the series by exactly 1, bidders would bid as if they are in a standard first-price auction with 2 bidders, i.e. $\beta(x) = E(Y_1|Y_1 < x)$. Since $Y_1 \equiv Y_1^{(N-1)}$ (the highest value of N-1 independent uniform draws), this expectation is equal to:

$$E(Y_1|Y_1 < x) = \left(1 + \frac{2-1+1-1}{2-1+1}(x-1)\right) = 1 + \frac{1}{2}(x-1) = \frac{1}{2} + \frac{1}{2}x$$

- Hence, we have:

$$\beta_1(x) = \frac{1}{2} + \frac{1}{2}x$$
$$\beta_2(x) = x$$

- Intuitively, bidders shade in the first auction since they have another chance of winning the good in the next auction. The bidders thus face a version of the standard trade-off between probability of winning vs. payoff if winning, since by increasing their bid they increase their chance of winning the item in the first auction but forego a potential payoff if could have won the item in the second auction where the bidder with the highest signal is out.
- The second auction inherits the standard second-price feature, where bidders bid truthfully since they have no extra chance of winning the item.

4b. What is the British government's total expected revenue, summed across the two auctions?

- In the second auction, expected revenue is equivalent to the expectation of the second-highest bid in the auction, which is equivalent to the second-highest value in the auction (as all bidder bid their value). In the second auction, the expectation of the second-highest value is equivalent to the expectation of the third-highest value across all three bidders (since the highest-value bidder will already have won the first auction), i.e. $E(R) = E(Y_3^3) = 1 + \frac{3+1-3}{3+1}(2-1) = \frac{5}{4}$.
- In the first auction, expected revenue is equivalent to the expected second-highest bid, which is equal to the bid placed by the bidder with the second-highest value:
$\frac{1}{2} + \frac{1}{2} * E(Y_2^3) = \frac{1}{2} + \frac{1}{2} * \left(\underline{x} + \frac{N+1-k}{N+1}(\overline{x} - \underline{x})\right) = \frac{1}{2} + \frac{1}{2} * (1 + \frac{3+1-2}{3+1}(2-1)) = \frac{1}{2} + \frac{1}{2} * \frac{3}{2} = \frac{5}{4}$.

- So total revenue is $\frac{5}{4} + \frac{5}{4} = \frac{10}{4}$.

A month before the auctions, DING gets a new CEO who is known as a bit of a wild-card. Specifically, she is a risk-lover.

4c. How does this development impact DING's bidding strategy in each of the two auctions? (a qualitative explanation is sufficient)

- In the second auction, this makes no difference – the standard second-price logic holds that it is always optimal to bid your value (regardless of risk preferences).
- In the first auction, a risk-lover would be willing to bid a little lower (i.e. shade a little more), for any given draw from their value distribution, in order to secure a higher potential profit (although sacrificing some probability of winning) – as this would be what the risk-loving bidder would have

done in the last of a series of first-price auctions. A potential bid strategy for DING could take the form e.g. $\beta_1(x) = \frac{2}{3} + \frac{1}{3}x$

Waterfall and Equisyd learn that DING's bidding strategy will be influenced by their new risk-loving CEO. Waterfall and Equisyd remain risk-neutral.

4d. How does the news about DING's preferences impact the bidding strategies of Waterfall and Equisyd (if at all)? (a qualitative explanation is sufficient)

- *Note: This question was difficult - a high grade was given for any submission which contained logical reasoning.*
- In the second auction, this makes no difference – the standard second-price logic holds that it is always optimal to bid your value (regardless of risk preferences).
- In the first auction, the knowledge that DING will bid differently will also impact the bidding strategies of Waterfall and Equisyd. Essentially, Waterfall and Equisyd will now also submit lower bids than they otherwise would have done (they will also shade more).
- The reasoning for this is that – for DING's highest possible valuation (a draw of 2) - DING will now submit a lower bid than they would have done had they been risk-neutral (given the result in 4c.). This means that DING's max. bid is lowered. For Waterfall and Equisyd, this information means that they will now not face the same extent of competition when they draw high values of x. Their original bid strategy prescribes bidding above DING's max. bid, i.e. in this range, they only face competition from each other, and not from DING. Given that this is the case, the trade-off between the probability of winning and profitability will change for Waterfall and Equisyd for high values of x. The probability loss associated with bidding a little lower will be less now that they do not face competition from DING on this interval, so the optimal balance between probability and profitability will be to bid lower (i.e. shade more) for higher values of x.

Problem 5

In 2013, Norway held a simultaneous multi-unit auction for three very important spectrum licenses.

There were three mobile network operators in Norway at the time (Telesyd, Telio and Tele3) that would all definitely take part at the auction, and there was also a small chance of a new entrant (i.e. a fourth bidder) taking part at the auction.

The regulator's primary policy objective associated with the allocation was to safeguard competition on the down-stream market for mobile services. The regulator was not directly concerned with revenue maximisation.

The Norwegian regulator decided to employ a discriminatory sealed-bid auction and set a reserve price of 0 for the licenses. The regulator also decided to impose a spectrum cap so that each bidder would be able to bid for a maximum of one license. Furthermore, the regulator decided not to disclose the number of qualified bidders prior to the auction.

5a. Imagine that you were in Tele3's position. What would have been your considerations in relation to bidding strategy? (a qualitative explanation is sufficient)

- As a point of departure, there were three likely bidders, three licenses, and a spectrum cap of one license per bidder. This meant that each of the three likely bidders could feel relatively certain that they would not face competition for the single license that they were allowed to bid for (since the two other likely bidders could only bid for max. two out of the three licenses, leaving one "up for grabs" for each bidder).

- Given the assumption that there would only be two other bidders at the auction, Tele3 would thus be tempted to submit a bid at the reserve price of 0 (or infinitesimally above 0), as this bid would be sufficient to win and would minimise Tele3's payment.
- However, if Tele3 expected a bid from a new entrant, then Tele3 would submit a larger bid. Depending on risk preferences, Tele3 would then submit a bid somewhere between 0 and their valuation (to balance the probability of outbidding at least one other bidder in order to win a license and profitability if the bid was successful). Since the spectrum licenses were very important, they might shade only a little in this case.
- Given that Tele3 did not know whether there would be two or three competitors, Tele3 faced a tricky problem of choosing between (or weighting) two potential bid strategies: one which dictated submitting a bid of 0 (if two competitors were expected) and one which dictated submitting a substantially higher bid (if three competitors were expected).
- If Tele3 felt very confident that only two competitors would participate, then Tele3's bidding strategy might be to bid 0.

The bidders submitted their bids and the regulator announced the results: The three licenses were allocated to Telesyd, Telio and a surprise 4^{th} bidder (a Russian oligarch), so Tele3 lost out.

Tele3 subsequently had to shut down its operations in Norway as it could not compete without winning one of the critical spectrum licenses. The number of competitors on the Norwegian market for mobile services thus decreased from three to two as a result of the outcome of the auction (at least until the Russian oligarch could manage to build up a network and enter the market). Competition was thus weakened, contrary to the regulator's policy objectives.

5b. What could the Norwegian regulator have done differently to avoid this outcome? Please provide several suggestions, if possible. (a qualitative explanation is sufficient)

- The regulator could have done several things differently in order to maximise the likelihood of all three established mobile network operators winning a license, thus safeguarding competition – or at least to maximise the probability of an efficient outcome. The regulator could have e.g.:
    - Employed a second-price auction format (e.g. a Vickrey) as this would have greatly simplified Tele3's bidding strategy – they would just submit a bid at their valuation. If they got lucky, and there was no new entrant, then they would pay 0. But if there was a new entrant, the pricing mechanism would force them to only just outbid the fourth highest bid. They would not have to worry about balancing profitability and probability under very uncertain circumstances, not knowing the number of bidders.
    - Employed a multi-round auction format (either SMRA or "simple clock"). Both formats would also greatly simplify bidding strategy considerations.
    - Released information about the number of qualified bidders prior to the submission of bids. This would have allowed Tele3 to optimally balance their bid, given the knowledge that they would be facing a bid from a new entrant – instead of constructing a bid under uncertainty regarding the extent of competition.
    - Employed a reserve price that would have been expected to be sufficient to deter new entrants from participating (new entrants would likely have lower valuations as their business case would be associated with far more start-up costs, uncertainty and initial investments).
    - Gotten rid of the spectrum cap altogether – or set it at two licenses instead of one. Although this would have allowed for situations where one of the three operators did not win a license because another operator won two, it would have forced all three bidders to bid more

aggressively for the first license, as they would know that they could not be certain of winning it – and this would likely have resulted in them winning one each.

- Not used an auction at all. If the regulator had a specific allocation in mind (one license for each of the three bidders), and did not want/need to generate revenues, then it does not make sense to employ an auction mechanism in the first place (auctions are designed for cases where the seller doesn't know the optimal allocation or price).